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CP-odd interaction of quarks and SU(2) gauge bosons with scalar field background in Kobayashi-Maskawa model.

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Abstract

CP-violating interaction of quarks and W-bosons with the scalar field background is studied in the Kobayashi-Maskawa with with Standard Model content of flavors and with additional heavy generation of fermions. The corresponding two-loop induced formfactors are calculated at zero temperature. The results are generalized at large momentum transfers to take into account CP-violating effects in the Higgs boson decay. The inclusion of this interaction into the scheme of adiabatic baryogenesis at the temperature of electroweak phase transition suffers from the uncertainties come from the poor knowledge of the vacuum condensate value triggering baryon number violating processes. It is shown, however, that even in the most favorable assumptions the Standard Model with four generations cannot produce enough C and CP violation for the explanation of the observable excess of baryons in the Universe.

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1 Introduction

The violation of CP symmetry observed 30 years ago [1] is still considered being an intriguing question of modern physics. The Kobayashi-Maskawa (KM) model is now the minimal explanation of this phenomenon in the framework of the Standard Model (SM). In this work we shall investigate in details the CP-odd interaction of the neutral scalar field with quarks and charged gauge bosons in KM model with the standard number of flavors and with additional heavy generation.

The search for CP-violating decays of Higgs boson into the pair of quarks or gauge bosons which may not be hopeless to see at future colliders has been investigated (theoretically) in various extensions of SM [2]. The KM model predictions for this processes was not considered up to now though. It is *a priori* clear that SM predictions for the amplitudes of interest are really tiny and therefore are of methodological interest only. The situation could be different when the theory contains one additional heavy fermion generation which existence is still allowed by current LEP data. Due to the absence of decoupling for heavy fermions in electroweak theory their influence on the low energy sector is significant. This work is aimed at the study of CP-violating amplitudes involving Higgs boson in the simplest extension of SM by new heavy generation of fermions with their dependence of unknown masses and mixing angles.

The second problem where the question of such interaction naturally arises is the baryogenesis at the temperature of the electroweak phase transition [3]. The promising feature of electroweak baryogenesis is in the possibility to explain the excess of baryons over antibaryons in the Universe without appealing to the GUT scale and staying in general on the background of known interactions. It is commonly understood, however, that the SM CP violation is capable to generate the asymmetry many orders of magnitude smaller than its experimentally observed value and this is the strong reason to look for a new CP-violating physics beyond SM.

The original analysis of the CP violation required for baryogenesis in KM model with three and four generations was done by Shaposhnikov [4]. Recently this problem has attracted serious attention again [5, 6, 7] in connection with the "chiral transport" scenario [8] according to which the baryoproduciton occurs in the vicinity of the narrow domain wall separating different phases. The opposite case of thick slowly moving wall allows for the "adiabatic" treatment of baryogenesis along the scenario considered first in Refs.[9, 10, 11, 13]. In this case the preferential production of baryons is governed by the effective C- and CP-odd interaction of quarks and $SU(2)$ gauge fields with the slowly varying vacuum expectation value (v.e.v.) of the scalar field. The magnitude of this interaction in KM model, being the matter of independent interest, may serve also as an additional check of the conclusions obtained in the general case [6, 7]. Models with one or several additional heavy generations deserve special analysis in this respect.

The organization of the article is following: Sections 2 and 3 contain the calculation of the CP-odd interaction of particles with Higgs background at zero temperature. Section 4 generalizes the results at $T \neq 0$. It comprises the estimate of the effect for the SM case and the calculation of the corresponding couplings for its four generation extension. In Appendix we extend the results obtained in Sections 3 at the case of large momentum

transfers to include the effects of CP violation in the decay of the real Higgs particle.

2 Flavor symmetry of amplitudes

It turns out that the two-loop induced CP-odd interaction discussed in [4] is easily calculable in zero temperature limit. Below we shall demonstrate that the effect is finite and satisfies all constraints imposed by the flavor interchange symmetry and V - A nature of charged currents.

The interaction of particles with the scalar field at small momentum transfer is commonly treated using the low energy theorem (See, for ex., the textbook [12]). It allows to obtain the amplitude of interest from the two-point self-energy function responsible for the propagation in the constant scalar field background $v+\chi$. Let us take, for example, the decay of the Higgs boson into two photons induced via one loop with heavy fermion, $m_f \gg m_{Higgs}$. The amplitude for this process could be easily reproduced as a first term of expansion in χ/v of the self-energy operator:

$$\frac{\alpha}{12\pi}\log\left(\frac{\Lambda^2}{(v+\chi)^2}\right)F_{\mu\nu}F^{\mu\nu} \longrightarrow -\frac{\alpha}{6\pi}\frac{\chi}{v}F_{\mu\nu}F^{\mu\nu}, \tag{1}$$

where Λ is the ultraviolet cutoff, χ is the Higgs boson field, v is the vacuum expectation value (v.e.v.) and $F_{\mu\nu}$ is the tensor of electromagnetic field. A similar attempt to apply this logic to the CP-odd interaction of quarks or gauge bosons with the scalar field background would stimulate one to look for the $(v+\chi)^2$-dependent part of corresponding CP-odd operators like $m_q\bar{q}i\gamma_5 q$ and $F^a\tilde{F}^a$. This dependence appears together with ultraviolet cutoff dependence. In SM it happens first in the fourteenth order of perturbation theory [14, 15]. We shall demonstrate, however, that already two-loop level is sufficient to induce the interaction of quarks and $SU(2)$ gauge bosons with the neutral scalar field background.

The CP violation in the model originates from the complexity of the KM matrix elements. For flavor-conserving amplitudes it manifests first in the quartic combination of KM matrices. The corresponding general diagrams are depicted at Fig. 1a and 1b. The solid line here denotes quark Green functions, wavy lines correspond to W-bosons. At the moment we choose the unitary gauge as containing the minimum of possible diagrams. These graphs are taken in the slow varying background of scalar field. The result could be presented in the form of the Effective Lagrangian as a series of operators with increasing power of space-time derivatives from this scalar field and we would keep only first non-vanishing terms. Since the Lorentz structure of the matrix element for the on-shell fermion and W-boson scattering off the scalar field is fixed:

$$M = A(q^2)i\bar{q}(p_1)\gamma_5 q(p_2) + B(q^2)\epsilon_{\alpha\beta\mu\nu}W^*_\alpha(p_1)p_{1\beta}W_\mu(p_2)p_{2\nu}, \tag{2}$$

we intend to calculate in fact the values of corresponding formfactors $A(q^2)$ and $B(q^2)$ at zero momentum transfer, $q^2=(p_2-p_1)^2=0$. Both these Lorentz structures vanish when $q_\mu \longrightarrow 0$ and therefore we cannot put $q=0$ from the very beginning. This ruins the naive approach based on the calculation of the two-point functions. Performing the calculation in the momentum representation we expand the amplitude over the small momentum q_μ

of the external Higgs boson and keep both zeroth and first order terms of the expansion. Technically this resembles at some points the analysis of the induced electric dipole moment (EDM) which is known to vanish to two loops [16].

First we determine the flavor arrangement along the fermion line. Let us denote by f the Green function of f-flavored fermion. Then a CP-odd amplitude for the quark scattering could be written in the following form:

$$\sum_{j,k,l} i\mathrm{Im}(V^*_{jf}V_{jk}V^*_{lk}V_{lf})\ fjklf. \tag{3}$$

For the fermionic loop at Fig. 1b the corresponding structure looks as:

$$\sum_{f,j,k,l} i\mathrm{Im}(V^*_{jf}V_{jk}V^*_{lk}V_{lf})\ fjkl, \tag{4}$$

where the cyclic permutation of the kind $fjkl = lfjk = klfj = jklf$ is allowed. It is easy to see that independently on the number of families the expressions (3) and (4) are antisymmetric under the interchange of flavors j and l:

$$\sum_{j,k,l} i\mathrm{Im}(V^*_{jf}V_{jk}V^*_{lk}V_{lf})\ fjklf = \frac{1}{2}\sum_{j,k,l} i\mathrm{Im}(V^*_{jf}V_{jk}V^*_{lk}V_{lf})\ f(jkl - lkj)f. \tag{5}$$

The extraction of the CP-odd part of the amplitude is simple when we deal with Standard Model with a sole imaginary phase. To be concrete we take a scattering of u quark in the scalar field background. Then the arrangement of flavors inside the loops is determined uniquely:

$$i\tilde{\delta}u\,(d(c-t)s - s(c-t)d + s(c-t)b - b(c-t)s + b(c-t)d - d(c-t)b)\,u, \tag{6}$$

where $\tilde{\delta} = \delta c_1c_2c_3s_1^2s_2s_3$ is the only possible CP-odd invariant of 3 by 3 KM matrix in standard parametrization [12].

The specific antisymmetrization of these amplitude in flavors causes, according to Shabalin [16], the identical cancellation of diagrams corresponding to electric dipole moments of quarks. The same is true for the EDM of W-boson and electron [17]. Therefore, we have to find out first wether the graphs determining the scattering in the scalar field background survive under the antisymmetrization in flavor.

The "dangerous" block responsible for the vanishing of EDMs comprises a mass operator (vertex part) between two fermion Green functions corresponding to different flavors. Taking into account all possible ways of external Higgs attachment, Fig. 2, we write down a general expression for this block:

$$\begin{aligned}\frac{\chi(q)}{v}\frac{1-\gamma_5}{2}\,[S_j(p-q/2)m_jS_j(p+q/2)M(p+q/2)S_k(p+q/2)&\\ +S_j(p-q/2)\Gamma(p,q)S_k(p+q/2)&\\ +S_j(p-q/2)M(p-q/2)S_k(p-q/2)m_kS_k(p+q/2)]\,\frac{1+\gamma_5}{2}\ -\ (j\leftrightarrow k),&\end{aligned} \tag{7}$$

where $S_j(p) = i(\hat{p} - m_j)^{-1}$ is the Green function of the j-flavored quark and $\hat{p} \equiv \gamma^\mu p_\mu$. $M(p)$ and $\Gamma(p,q)$ are the one-loop induced mass operator and vertex part respectively.

It should be mentioned here that other possibilities of the external Higgs attachment, to external fermion lines or to the outer W-propagator, are not operative due to the identity:

$$\frac{1-\gamma_5}{2} S_j(p) M(p) S_k(p) \frac{1+\gamma_5}{2} \; - \; (j \leftrightarrow k) \equiv 0. \tag{8}$$

The V-A character of charged currents fixes the general structure of the mass operator before renormalization up to an invariant function depending on p^2:

$$M = \hat{p} \frac{1-\gamma_5}{2} f(p^2). \tag{9}$$

The on-shell renormalization with respect to quark j from the left and quark k from the right introduces into the mass operator the dependence of external masses [16, 17]:

$$M_r = \tilde{f}(p^2) \hat{p} \frac{1-\gamma_5}{2} - f_{jk} [\hat{p} \frac{1+\gamma_5}{2} - m_j \frac{1+\gamma_5}{2} - m_k \frac{1-\gamma_5}{2}], \tag{10}$$

where f_{jk} and $\tilde{f}$ are expressed via the function f and masses $m_j, \; m_k$ as follows:

$$\tilde{f}(p^2) = f(p^2) - \frac{m_j^2 f_j - m_k^2 f_k}{m_j^2 - m_k^2}, \quad f_{jk} = \frac{m_j m_k (f_j - f_k)}{m_j^2 - m_k^2}; \;\; f_j = f(p^2 = m_j^2). \tag{11}$$

For the simplicity we use the nonrenormalized form of the mass operator and then show that the same result remains intact for the full renormalized expression.

The expansion in q_μ is essential at the next step of the calculation. To proceed forward with it we connect the vertex part at zero momentum transfer, $\Gamma(p, q = 0)$, to the nonrenormalized mass operator using the Ward identity:

$$\Gamma(p, q = 0) = \frac{\partial}{\partial v} M(p). \tag{12}$$

It is worth to note that the unitary gauge ensures the cancellation of all divergencies in Γ. Moreover, this vertex corresponds to the operator of dimension 5, $\chi \bar{q}_j \hat{\partial} (1 - \gamma_5) q_k$, and does not require renormalization.

The zeroth order term of the expansion of expression (7) in q vanishes simply because it could be reduced to the total derivative $\partial/\partial v$ from the l.h.s. of the identity (8). The same is true and for any order in χ/v if we systematically neglect the momentum associated with the Higgs field.

The first term of the expansion in q does not vanish, however. After a straightforward arithmetic we get:

$$- i4 \frac{\chi(q)}{v} \frac{(m_j^2 - m_k^2) \hat{p} (pq)}{(p^2 - m_j^2)^2 (p^2 - m_k^2)^2} \left(p^2 \frac{\partial f}{\partial p^2} + \frac{v}{2} \frac{\partial f}{\partial v} \right) \frac{1+\gamma_5}{2}. \tag{13}$$

The interesting feature of the formula (13) consists in the vanishing of the expression in parenthesis for any given function depending on the ratio p^2/v^2:

$$\left(p^2 \frac{\partial}{\partial p^2} + \frac{v}{2} \frac{\partial}{\partial v} \right) f(p^2/v^2) \equiv 0 \tag{14}$$

Despite appearance the effect is not zero due to logarithmically divergent part of the mass operator. Taking into account an explicit dependence of the ultraviolet cutoff Λ we obtain:

$$\left(p^2\frac{\partial}{\partial p^2}+\frac{v}{2}\frac{\partial}{\partial v}\right)\log\frac{\Lambda^2}{p^2+v^2}=-1. \tag{15}$$

Let us demonstrate this assertion in more details. First we note that the GIM property makes the integral defining function f be almost convergent and therefore be dependent on the ratio p^2/v^2. The word "almost" refers to the only possible logarithmically divergent term which originates from the longitudinal part of the W-boson propagator:

$$\begin{aligned} f\hat{p}\frac{1-\gamma_5}{2} &= -i\int\frac{d^4q}{(2\pi)^4}\frac{m_t^2\hat{q}(\hat{p}+\hat{q})\hat{q}}{M_w^2((p+q)^2-m_t^2)(p+q)^2q^2}\frac{1-\gamma_5}{2} \\ &= \frac{3g_w^2m_t^2}{4M_w^2}\frac{1}{16\pi^2}\log\frac{\Lambda^2}{p^2+v^2}\hat{p}\frac{1-\gamma_5}{2}+..., \end{aligned} \tag{16}$$

where we have imposed without the lost of generality the obvious relation $m_t^2 \gg m_c^2$. In use of formulae (15) and (16) the resulting expression is transformed to the following form:

$$-i\frac{\chi(q)}{v}\frac{(m_j^2-m_k^2)\hat{p}(pq)}{(p^2-m_j^2)^2(p^2-m_k^2)^2}\frac{3g_w^2m_t^2}{M_w^2}\frac{1}{16\pi^2}\frac{1+\gamma_5}{2}. \tag{17}$$

The same answer emerges as the result of calculation with the renormalized mass operator M_r. The contribution from the counterterms in (10) vanishes since it is symmetric under the interchange of m_i and m_j. We skip here the prove of this statement which is rather simple.

To conclude this section, we have shown that the antisymmetry under the interchange of flavors does not lead to the vanishing of the amplitude of interest. However, strong cancellations exist between mass operator and vertex part contributions which effectively reduces the whole calculation to the one-loop level. The inner loop produces just a constant multiplier proportional to the square of the fermion Yukawa coupling on account of Eqs. (15) and (16). We performed also the calculation in the Landau gauge, $\xi = 0$, where the answer originates from the diagrams with charged Higgs bosons. After the complete summation over flavors the results of calculations in different gauges coincide identically.

3 KM predictions for the formfactors

Using the results of the previous section it is easy to integrate over the second loop and find corresponding amplitudes. Now it is convenient to treat SM model and its heavy quark extensions separately.

1. Standard Model set of flavors
It is easy to see that the interaction of the scalar field with u and c quarks in SM is much larger than with other flavors. It is simply explained by the m_t^2-enhancement factor for

the interaction with external c- or u-flavored quarks. The sum over flavors and the loop integral are trivial so the final answer reads as follows

$$\mathcal{L}_{eff} = -\frac{3}{32\pi^4}\tilde{\delta}\frac{G_F}{\sqrt{2}}f_t^2 m_s^2 \log\frac{m_b^2}{m_s^2}\,\frac{\partial_\mu \chi}{v}\left(\bar{c}\gamma_\mu\frac{1-\gamma_5}{2}c - \bar{u}\gamma_\mu\frac{1-\gamma_5}{2}u\right). \tag{18}$$

Here we introduce the Fermi constant, $G_F = \sqrt{2}g^2/(8M^2)$, and the Yukawa coupling of fermion $f_i = m_i/v$ in a standard way. The integral is calculated to logarithmic accuracy and m_s^2 at lower limit rather symbolizes a momentum scale of order Λ_{QCD}. Integrating by parts and applying the equation of motion for external quarks we reduce the operator structure of (18) to the common form $i\chi m_i \bar{q}_i\gamma_5 q_i$. For down quarks (d and s) the result is proportional to the combination $f_b^2 m_c^2$. The CP-odd interaction of third generation quarks with scalar field acquires even stronger suppression.

The answer (18) is valid when the momentum transfer q does not exceed m_s. It is clear, however, that there is an easy way to generalize the answer at large values of q^2 using m_t^2-dependence of the answer. The inner loop behaves as an effective constant vertex until $|q^2|$ becomes comparable with m_t^2. Therefore, the amplitude of interest may serve not only for the scattering of quark in the scalar field background but also for the decay of the real Higgs into the quark-antiquark pair if scalar boson is not very heavy. The generalization at large momentum transfers of the interaction (18) and of other results from this section are accumulated in the Appendix.

Going over the calculation of the Higgs-W-W interaction we determine first the flavor structure of the fermionic loop. Its CP-odd part is given by the following combination:

$$\begin{aligned}
&i\tilde{\delta}[d(c(b-s)t - t(b-s)c + t(b-s)u - u(b-s)t + u(b-s)c - c(b-s)u)\\
&+s(c(d-b)t - t(b-s)c + t(d-b)u - u(d-b)t + u(d-b)c - c(d-b)u)\\
&+b(c(s-d)t - t(s-d)c + t(s-d)u - u(s-d)t + u(s-d)c - c(s-d)u)]
\end{aligned} \tag{19}$$

Each product of four quark Green functions allow for the cyclic permutation of the kind:

$$udcs = dcsu = csud = sudc.$$

The "degree of antisymmetry" of eq. (19) is higher than that of corresponding structure with external fermions (6). This results on the stronger suppression of the interaction of W boson with external scalar field:

$$\mathcal{L}_{eff} = \frac{3}{32\pi^4}\tilde{\delta}\frac{G_F}{\sqrt{2}}\frac{f_t^2 m_c^2 m_s^2}{m_w^2}\log\frac{m_b^2}{m_s^2}\,\frac{\chi}{v}\epsilon_{\alpha\beta\mu\nu}\partial_\alpha W_\beta \partial_\mu W_\nu^*, \tag{20}$$

where $W_\beta = (\sqrt{2})^{-1}(W_\beta^1 + iW_\beta^2)$. The calculation is performed for on-shell W-bosons. The leading contribution to (20) comes again from the top quark flowing inside the inner loop at Fig. 1b.

2. KM model with the additional heavy generation(s)

The consideration of the SM prediction for the CP-odd interaction with scalar field is mostly of methodological meaning. The resulting amplitudes are too small to produce any observable effects. Now we shall extend SM by adding a new heavy generation with standard quantum numbers preserving the same KM origin of CP violation. The phenomenological

constraints on the parameters of this model are provided by the analysis of K and B meson mixing [18] and electroweak precision data [19]. When the mass of 4th generation is large and lies somewhere between 500Gev and 1Tev one comes to the picture of the strongly interacting Higgs-fermion sector [20]. We assume that masses of h and g quarks are of order 500Gev preserving the perturbative unitarity.

The 4×4 KM matrix possesses three independent CP-odd invariants. The dynamical enhancement of flavor-diagonal CP-violating amplitudes are associated with the invariant corresponding to the mixing of second, third and fourth generations of quarks [21]. The source of this enhancement is in the change of the overall mass factor in nominators of formulae (18)-(20).

Let us demonstrate this assertion on the example of the s and b quarks scattering off the Higgs background. Instead of SM prediction with the dependence of $m_b^2 m_c^2$, we may expect the effect in the four generation model to arise with a factor $m_g^2 m_t^2$, and the total enhancement could reach 10^8. Large masses in nominator imply that the characteristic loop momenta are also large. Therefore, inside the loops, we are legitimate to put all quark masses to zero except m_t, m_g and m_h. In other words, inside the loops, we are able to identify propagators of light quarks:

$$c = u \equiv U; \quad d = s = b \equiv D.$$

After that to sufficient accuracy we derive the flavor structure of the amplitude for the b quark interaction with Higgs background (See Ref.[21] for details):

$$i\mathrm{Im}(V_{ts}^* V_{tb} V_{hb}^* V_{hs}) b[t(g-D)h - h(g-D)t + U(g-D)t - t(g-D)U + h(g-D)U - U(g-D)h]b \tag{21}$$

The rephasing invariant combination of KM matrix elements in (21) to good accuracy coincides with that responsible for CP-odd B_S^0 meson mixing. In terms of Wolfenstein parameter it could naturally reach the order

$$\mathrm{Im}(V_{ts}^* V_{tb} V_{hb}^* V_{hs}) \sim \lambda^5. \tag{22}$$

In the outer loop integral the main contribution now comes from the longitudinal part of W-propagator. For the on-shell quarks the result is presented again in the form of the effective Lagrangian:

$$\mathcal{L}_{eff} = \mathrm{Im}(V_{ts}^* V_{tb} V_{hb}^* V_{hs}) \frac{3}{128\pi^4} f_g^2 f_t^2 (\log \frac{m_h^2}{m_t^2} - 1) \, \frac{\chi}{v} \left(m_b \bar{b} i\gamma_5 b - m_s \bar{s} i\gamma_5 s \right), \tag{23}$$

where to sufficient accuracy we have omitted m_w^2/m_t^2 suppressed terms. We should take into account, of course, the range of validity of perturbative analysis in this model. When the Yukawa constants become sufficiently bigger than unity the perturbative expansion does not work and we have to deal with a strong coupling regime. In our case, however, all "dangerous" vertices are proportional to the combination of $V_{hj} f_h$ or $V_{jg} f_g$. According to Refs. [18, 21] we take non diagonal KM matrix elements not exceeding λ and therefore we could extend the perturbative analysis until $f_{h(g)} \sim \lambda^{-1}$.

The interaction with W-boson in this model is also enhanced in comparison with SM case. However, our approximation with a complete degeneracy between light quarks inside

the loops is not operative for the interaction with W-boson because the amplitude (4) vanishes in this limit. To obtain a nonvanishing effect we must take into account the mass of b quark and to that reason it is the m_b^2-suppressed effect:

$$\mathcal{L}_{eff} = -\mathrm{Im}(V_{ts}^* V_{tb} V_{hb}^* V_{hs}) \frac{3}{32\pi^4} \frac{G_F}{\sqrt{2}} m_b^2 \log \frac{m_w^2}{m_b^2} \frac{\chi}{v} \epsilon_{\alpha\beta\mu\nu} \partial_\alpha W_\beta \partial_\mu W_\nu^*. \tag{24}$$

4 Generalization at nonzero temperature and adiabatic baryogenesis

The interest to the CP-odd interaction of quarks and W-bosons with a nonuniform scalar field background is mainly inspired by the problem of systematic change of baryon number during the electroweak phase transition. The results of previous sections cannot be directly used in this context because they deserve a considerable modification to meet the high temperature conditions. Before doing that we would like to remind some principal points of electroweak baryogenesis without going into details and following in general the reviews [22, 23].

It is commonly understood now that the observed ratio of baryon to photon densities in the Universe,

$$\frac{N_B - N_{\bar{B}}}{N_\gamma} \sim 10^{-10}, \tag{25}$$

could be achieved, in principle, during the electroweak phase transition. Moreover, the SM content of fields may generate all three Sakharov's conditions [24] necessary for baryogenesis.

The first criterion of the microscopic baryon number nonconservation is fulfilled due to the anomaly in the current associated with this number:

$$\partial_\mu j_b^\mu = \frac{3\alpha_w}{8\pi} F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a. \tag{26}$$

The r.h.s. of this equation is the total derivative, $\partial_\mu K_\mu$, and it implies that baryon number conserves in the specific combination with the topological charge, $\Delta(Q_b - Q_{cs}) = 0$, where by definition

$$Q_{cs} = \int d^3x K_0 = \frac{g^2}{16\pi^2} \epsilon^{ijk} \int Tr \left(F_{ij} W_k + \frac{2}{3} ig W_i W_j W_k \right) d^3x. \tag{27}$$

Here $W_i = W_i^a \tau_a/2$ and $F_{ij} = \partial_i W_j - \partial_j W_i + ig[W_i, W_j]$. At zero temperatures the effects of tunneling between topologically distinct vacua are exponentially suppressed. In contrast to that at very high temperature in the unbroken phase the exponential suppression is removed and the rate of the processes with $\Delta Q_{cs} = \Delta Q_b \neq 0$ is believed to go as

$$\Gamma = c\alpha_w^4 T^4, \tag{28}$$

where c is some dimensionless unknown coefficient. With the growth of the scalar field v.e.v. the exponential suppression is switch on at some value v_0 which is not known to sufficient accuracy. Simple arguments suggest the order of magnitude estimate for this value:

$g_w v_0 \sim \alpha_w T$, whereas the semiclassical analysis of sphaleron processes gives a numerical enhancement for this value: $g_w v_0 \sim 14\alpha_w T$ [25].

The second requirement could also be satisfied. The analysis of the effective potential at the critical point suggests the possibility of the first order phase transition between symmetric, $v = 0$, and broken, $v = v_c \neq 0$, phases. The propagation of the domain walls separating two phases through the relativistic plasma breaks the thermal equilibrium and generates the arrow in time. After the transition all processes with $\Delta Q_b \neq 0$ should be suppressed, i.e. $v_c > v_0$, to avoid the baryon number erasure [3]. The C and CP violation shifts the processes with $\Delta Q_b \neq 0$ toward a preferential production of baryons. However, the amount of CP violation inherently presented in SM could lead to an asymmetry, according to Refs.[6, 7], of order $n/s \sim 10^{-27}$ as best and this is the main obstacle for SM explanation of the baryon number of the Universe. In what follows we concentrate ourself on the analysis of CP violation developed in KM model with different numbers of generations assuming that all other conditions of baryoproduction are indeed satisfied.

When the domain wall is thick one comes to the "adiabatic" treatment of baryogenesis [10, 11, 13]. In this case, considerably simplified in comparison with the generic situation, the slowly varying in time vacuum expectation value of the scalar field serves as a chemical potential for the Chern-Simons charge:

$$L_{int} = A\dot{v}Q_{cs} \tag{29}$$

The appropriate sign of A and B generates the arrow for the sphaleron processes toward the observed density of baryons. Then the total amount of baryons could be estimated as follows [23]:

$$Q_b \sim \frac{1}{T}\int \Gamma A\dot{v}\, dt \tag{30}$$

There are two different scheme to generate CP-violating operators analogous to (29) in the model. First one refers to the case of small vacuum expectation values of the scalar field and could be achieved through the many loop mechanism like that proposed in different context by Ellis and Gaillard [14] and readdressed for baryogenesis by Shaposhnikov [4] (See also the review [23]). The growth of v.e.v. suggests the switch to another regime [4] where the low-loop amplitudes are less suppressed. We estimate the critical field v_1 where the change of these regimes occurs as following:

$$\frac{v_1^2}{(\pi T)^2} \sim \frac{1}{16\pi^2} \tag{31}$$

The v.e.v. of the scalar field is normalized at $(\pi T)^2$ as it follows from the finite temperature Feynman rules. It is quite possible also that $v_1 < v_0$ i.e. this value develops *after* all sphaleron-like processes become suppressed and therefore the two-loop induced interaction does not affect the baryon density. The opposite case of $v_1 > v_0$ deserves special consideration.

1. Standard Model set of flavors

If the baryon number violating processes shut down at sufficiently large v.e.v. it is reasonable to generalize the two loop mechanism described in Sections 2 and 3 at nonzero

temperatures. First, we note that in any case the result expected is really tiny and there is no need in the exact calculation. The calculation itself is now more complicated than in the case of $T = 0$ due to the existence of another dimensional parameter T and the lack of Lorentz invariance. Thus, simple arguments leading to the elimination of the momentum dependence come from the inner loop do not work and we have to deal with a real two-loop calculation.

Let us integrate over all quark fields. Then the effective operators governing baryoproduction could be composed from neutral scalar and vector boson fields. It is clear that in leadin order they originate from the two loop diagram at Fig.1b. The quark operators contribute to the effect at the next three-loop order and thus are neglected.

The quark mass dependence of the answer arises through the mass insertions [4] affecting the stronger compensation of different diagrams than it happens at $T = 0$. At the first glance the flavor structure (19) of the fermionic loop implies that the effect arises first in the v^{10}-order together with antisymmetric product of Yukawa couplings:

$$(f_t^2 - f_c^2)(f_t^2 - f_u^2)(f_c^2 - f_u^2)(f_b^2 - f_s^2)(f_b^2 - f_d^2)(f_s^2 - f_d^2) \simeq f_t^4 f_c^2 f_b^4 f_s^2 \tag{32}$$

Two powers of Yukawa coupling originate here from the vertices with charged Higgs and are not accompanied by v^2. However, the actual degree of suppression is even stronger. The expression (19) changes the sign under the permutation of up and down families of flavors whereas the lowest order dependence of Yukawa couplings (32) is explicitly symmetric. It means that in this order the diagrams with up quarks flowing inside mass operator cancel those with down quarks. To avoid this cancellation one has to introduce the weak isospin asymmetry via an additional loop with the exchange of U(1) gauge boson or via two additional mass insertions for the top flavor. The second possibility gives bigger interaction in the chosen conditions when the tree level dominates over loop corrections. Thus, the resulting estimate takes the form:

$$L_{int} \sim \tilde{\delta}\frac{3}{8\pi^2} f_t^6 f_c^2 f_b^4 f_s^2 \frac{\dot{v}v^{11}}{(\pi T)^{12}} \frac{g^2}{16\pi^2}\epsilon^{ijk}\int (W_i^{(1)}\partial_j W_k^{(1)} + W_i^{(2)}\partial_j W_k^{(2)})d^3x \tag{33}$$

This interaction does not literally correspond to the form (29). We see that it depends on the global orientation n_a of the vacuum configuration of the scalar field doublet ϕ:

$$\partial_\alpha W_\beta^1 \partial_\mu W_\nu^1 + \partial_\alpha W_\beta^2 \partial_\mu W_\nu^2 = \partial_\alpha W_\beta^a \partial_\mu W_\nu^a - n^a n^b \partial_\alpha W_\beta^a \partial_\mu W_\nu^b \tag{34}$$

where

$$v^2 n^a \equiv \phi^\dagger \tau^a \phi.$$

We would assume that the nucleation of the new phase occurs with random orientation of n^a in different bubbles. Simple average over this orientation gives the coefficient 2/3. The absence of terms trilinear in gauge field in the expression (33) does not mean that they cannot be induced or do not affect sphaleron-like processes. Since the $SU(2) \times U(1)$ symmetry is spontaneously broken we could regard quadratic and trilinear terms as independent operators. It is clear that any nonorthogonal to Q_{cs} linear combination of these operators generates an effective chemical potential for Q_b. Combining several factors we estimate the resulting asymmetry to arise at the level:

$$\frac{N_B - N_{\bar{B}}}{N_\gamma} \sim 10^{-2}\alpha_w^4 \frac{3}{8\pi^2}\tilde{\delta} f_t^6 f_c^2 f_b^4 f_s^2 \frac{v^{12}}{(\pi T)^{12}} \tag{35}$$

Numerically this asymmetry is really tiny and reaches 10^{-40} in the most optimistic assumptions about the sphaleron cuttoff.

2. KM model with the additional heavy generation(s)
The estimate of the effect in this case goes along the same way if we take the unknown heavy masses somewhere around the mass of the top quark, $m_{h(g)} \sim m_t$. Thus, simply renaming quarks and phases in the expression (33) and taking into account the isospin splitting inside third generation we obtain:

$$L_{int} \sim \mathrm{Im}(V_{ts}^* V_{tb} V_{hb}^* V_{hs}) \frac{3}{8\pi^2} f_h^4 f_t^4 f_g^4 f_b^2 \frac{\dot{v} v^{11}}{(\pi T)^{12}} \frac{g^2}{16\pi^2} \epsilon^{ijk} \int (W_i^{(1)} \partial_j W_k^{(1)} + W_i^{(2)} \partial_j W_k^{(2)}) d^3x. \tag{36}$$

The case of very heavy h and g quarks, $m_{h(g)}^2 \gg m_t^2$, requires separate treatment because at some v.e.v. v_2 their masses become comparable with temperature, $f_h v_2 \sim T$, and cannot be treated as mass insertions. The simplification of the calculation is possible if we take $f_h v \gg T$. It allows to use zero temperature expressions for the inner loop with quarks from the fourth generation and calculate the rest of diagram with "light quarks" in accordance with the finite temperature technique. After the straightforward calculation we get:

$$\begin{aligned} L_{int} = \mathrm{Im}(V_{ts}^* V_{tb} V_{hb}^* V_{hs}) \frac{3}{32\pi^2} \lambda(5)(f_h^2 - f_g^2) f_t^2 f_b^2 \frac{\dot{v} v^3}{(\pi T)^4} \frac{g^2}{16\pi^2} \times \\ \epsilon^{ijk} \int (W_i^{(1)} \partial_j W_k^{(1)} + W_i^{(2)} \partial_j W_k^{(2)}) d^3x, \end{aligned} \tag{37}$$

where $\lambda(5) = \frac{31}{32}\zeta(5) \simeq 1$.

It is instructive to compare the size of this interaction with the CP-even coupling of the topological charge and the relative phase of different scalars which arises in the multi Higgs models [11]. This effective interaction governs baryoproduction in the model when the CP-violating phase in Higgs sector changes from zero to some finite value θ. Being generated at one-loop level, this coupling is proportional to $m_t^2\theta$ and this is again v^4-dependence [23]. We would like to stress that the relative smallness of the effect in KM model with four generation of fermions results from:
a) Flavor symmetry of the amplitude and corresponding f_b^2-suppression;
b) Smallness of the CP-odd angle invariant $\mathrm{Im}(V_{ts}^* V_{tb} V_{hb}^* V_{hs})$. It could be restricted from the data on neutral B_S meson mixing and is unlikely to exceed λ^5 in terms of Wolfenstein parameter;
c) Additional limits on the mass difference of heavy quarks come from electroweak ρ parameter. Both f_h^2 and f_g^2 could be rather large. However, the analysis of electroweak precision data implies that h and g quarks must be sufficiently degenerate in masses, $(f_h - f_g)^2 \ll 1$. So, the factor $f_h^2 - f_g^2 \simeq 2f_h(f_h - f_g)$ does not exceed unity if we take $m_{h(g)}$ somewhere in the interval 500 Gev - 1 Tev.

If we would introduce into the theory two or more additional heavy generations, (h, g); (h', g');..., with a large mixing between them we would remove f_b^2-dependence. At the same time there is no strict limits on the mixing between heavy generations and we could expect the corresponding CP-odd combination of mixing angles $\mathrm{Im}(V_{tg'}^* V_{tb} V_{hb}^* V_{hg'})$ to be large. The third factor of suppression related to the antisymmetry under the interchange of U and D types of quarks is still held. Now the CP violation occurs entirely in fermion-Higgs sector

of the theory and it is clear that potentially the KM type of models with two or more heavy generations of quarks are capable to produce the amount of CP-violation comparable with that of multi-Higgs models. Unfortunately, the simplest variant of this model is already excluded by electroweak precision data analysis.

5 Conclusions

We have shown that the two-loop level is sufficient to to induce the CP-odd interaction of particles with the scalar field background in the KM model. Together with the magnetic quadrupole moment of the W-boson [27] these are the only known examples of flavor-conserving CP-odd operators of low dimension, dim$\leq$6, which do not vanish to two-loop approximation in this model in zero temperature limit. The sum over flavors inside the loops leads to the remarkable compensation of different contributions which affects in its turn on the effective simplification of the whole calculation. The inner loop gives just a constant multiplier proportional to the square of the Yukawa coupling of the fermion flowing inside this loop. This is an explicit example of the nondecoupling of heavy fermions in the electroweak theory: for the SM set of flavors the result is proportional to f_t^2, for its four generation extension it is $f_{h(g)}^2$, etc. The absence of decoupling allows to extend the calculation at large momentum transfers and keep the effective one-loop level of difficulty. The resulting amplitudes describes the CP violation in the decay of the Higgs boson to accuracy $\mathcal{O}(m_{Higgs}^2/m_f^2)$, where m_f is the mass of the heaviest quark.

The attempt to plug this interaction into the scheme of electroweak baryogenesis at the temperature of the phase transition faces with the poor knowledge of the vacuum expectation value v at which all transitions with $\Delta Q_b \neq 0$ become suppressed. In the assumption that this value is rather large, $g_w v$ is of order several units of $\alpha_w T$, the two-loop CP-violating mechanism dominates over multi-loop ones. The prescriptions of the finite temperature diagram technique together with the flavor symmetry of diagrams provide a strong suppression of the interaction of interest. From naive expectations in the SM case the effect is proportional to the minimal CP-odd combination of mixing angles and Yukawa couplings $\tilde{\delta} f_t^4 f_c^2 f_b^4 f_s^2 v^{10} (\pi T)^{-10}$. The additional antisymmetry with respect to interchange of U and D types of quarks makes the total degree of suppression be even smaller: $\tilde{\delta} f_t^6 f_c^2 f_b^4 f_s v^{12} (\pi T)^{-12}$. The analysis of the KM model with four generations of quarks is performed in the most interesting situation when the heaviest quark masses are comparable with temperature. The corresponding size of the effect now is: $\mathrm{Im}(V_{ts}^* V_{tb} V_{hb}^* V_{hs})(f_h^2 - f_g^2) f_t^2 f_b^2 v^4 (\pi T)^{-4}$.

All three factors of suppression pointed out in the previous section will be held in a generic nonadiabatic case. The numerical raise of the effect in the presence of the narrow wall is connected mainly with the change of temperature infrared cutoff. Instead of powers of πT in the denominator of formulae (33)-(37) one could expect the normalization on parameters characterizing the propagation and collisions of quasiparticles in the hot plasma $\sim g_s T$.

Our intention to make KM type of CP violation useful for electroweak baryogenesis implies to introduce two new heavy generations at least. The amount of CP violation developed at high temperatures in this model does not differ considerably from that of

very popular multi-Higgs extensions of SM. To make this model be consistent with electroweak precision data constraints on isospin symmetric observables [19] one has to introduce additional bosons into the theory to compensate the large positive contribution to the S-parameter. Another weak point of the proposed analysis is in the neglection of other requirements needed for baryoproduction. It is clear that heavy fermions affect the character of the phase transition. In the perturbative treatment of the effective potential they weaken the first order phase transition. To avoid the baryon number erasure after the transition and ensure the $T = 0$ Higgs boson mass to satisfy modern experimental limits one comes again to the necessity of additional bosons in the theory [26]. This would lead in a generic situation to new sources of CP violation besides complexity of the KM matrix.

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APPENDIX

The generalization of static formfactors at large momentum transfers is performed below in use of the f^2 dependence of the one-loop Higgs-fermion-fermion vertex. The characteristic momentum inside this loop is large and determined by the m^2, the mass of the heaviest fermion. This allows us to take this vertex in the form:

$$M = \frac{\chi}{v}\frac{3f^2}{8\pi^2}(\hat{p}_1 + \hat{p}_2)\frac{1-\gamma_5}{2} + \mathcal{O}(\frac{m_w^2}{m^2}; \frac{q^2}{m^2}; \frac{p_1^2}{m^2}; \frac{p_2^2}{m^2}), \tag{38}$$

where we used again the unitary gauge. Besides the trivial kinematic factor $\hat{p}_1 + \hat{p}_2$ of incoming and outgoing fermion momenta this vertex is momentum independent and therefore the whole computation is effectively of one-loop difficulty level.

In the SM the growth of $|q^2|$ brings the additional serious suppression of the interaction of interest. Now the CP-violating amplitude reads as follows:

$$\begin{aligned} M &= -\frac{3}{32\pi^4} f_t^2 \frac{G_F}{\sqrt{2}} \frac{m_s^2 m_b^4}{q^4} \log\frac{|q^2|}{m_b^2}\ \frac{\chi}{v}(m_c\bar{c}i\gamma_5 c - m_u\bar{u}i\gamma_5 u) \quad \text{at} \quad m_b^2 \ll |q^2| \ll m_w^2; \\ M &\simeq \frac{9}{128\pi^4} f_t^2 \frac{G_F}{\sqrt{2}} \frac{m_s^2 m_b^4}{q^2 m_w^2} \log\frac{m_w^2}{m_b^2}\ \frac{\chi}{v}(m_c\bar{c}i\gamma_5 c - m_u\bar{u}i\gamma_5 u) \quad \text{at} \quad m_w^2 \ll |q^2| \ll m_t^2, \end{aligned} \tag{39}$$

where we hold only $\log m_b^2$-contributions, i.e. infrared enhanced terms. We see the restoration of the factor m_b^4, not unlike it happens at high temperature. We skip here the calculation of the interpolation between these two formulae at $q^2 \sim m_w^2$ which is also, of course, very simple.

The W-boson interaction with Higgs drops with the growth of q^2 as follows:

$$\begin{aligned} M &\simeq \frac{3}{32\pi^4}\tilde{\delta}\frac{G_F}{\sqrt{2}}\frac{f_t^2 m_c^2 m_b^4 m_s^2}{q^4 m_w^2}\log\frac{|q^2|}{m_w^2}\ \frac{\chi}{v}\epsilon_{\alpha\beta\mu\nu}\partial_\alpha W_\beta \partial_\mu W_\nu^*, \\ & \qquad \text{at} \quad m_w^2 \ll |q^2| \ll m_t^2, \end{aligned} \tag{40}$$

where from the reasons of simplicity we hold only contributions proportional to $\log m_w^2$.

The four generation case allows for the generalization at larger momenta. For the physically interesting scale relation $m_w^2 \ll |q^2| \sim m_t^2 \sim |q^2 - m_t^2| \ll m_{h(g)}^2$ we obtain the following result:

$$M = \mathrm{Im}(V_{ts}^* V_{tb} V_{hb}^* V_{hs}) \frac{3}{128\pi^4} f_g^2 f_t^2 (\log \frac{m_h^2}{m_t^2} + \frac{m_t^2 - q^2}{q^2} \log \frac{m^2}{|m^2 - q^2|} - 2) \times \frac{\chi}{v} \left(m_b \bar{b} i\gamma_5 b - m_s \bar{s} i \gamma_5 s \right), \tag{41}$$

We see that the ratio of the CP-odd couplings to CP-even ones for s and b flavors is parametrically suppressed in fact only by the combination of mixing angles and does not drop with the growth of q^2. Unfortunately, the size of this amplitude, being considerably enhanced in comparison with SM predictions, is not likely to be observed mainly because of the smallness of mixing angle combination. Finally, the CP-odd interaction of W-boson with Higgs reads as follows:

$$M \simeq -\mathrm{Im}(V_{ts}^* V_{tb} V_{hb}^* V_{hs}) \frac{3}{32\pi^4} \frac{G_F}{\sqrt{2}} \frac{f_h^2 m_b^2 m_w^2}{q^2} \log \frac{|q^2|}{m_w^2} \left(2 + \frac{m_t^2}{q^2} \log \frac{|q^2 - m_t^2|}{m_t^2} \right) \times \frac{\chi}{v} \epsilon_{\alpha\beta\mu\nu} \partial_\alpha W_\beta \partial_\mu W_\nu^* . \tag{42}$$

References

[1] J.H. Christenson *et al.*, Phys. Rev. Lett. **13** (1964) 138.

[2] B. Grzadkowski and J.F. Gunion, Phys. Lett. **294** (1992) 361; D. Chang, W.-Y. Keung and I. Phillips, Phys. Rev. **D48** (1993) 3225; A. Soni and R.M. Xu, Phys. Rev. **D48** (1993) 5259; X.-G. He, J.P. Ma and B. McKellar, Preprint UM-P-93/89.

[3] V.A. Kuzmin, V.A. Rubakov, M.E. Shaposhnikov, Phys. Lett. **B155** (1985) 36.

[4] M.E. Shaposhnikov, Nucl. Phys. **B287** (1987) 757.

[5] G. Farrar and M. Shaposhnikov, Phys. Rev. Lett. **70** (1993) 2833.

[6] M.B. Gavela *et al.*, Nucl. Phys. **B430** (1994) 345, Nucl. Phys. **B430** (1994) 382.

[7] P. Huet and E. Sather, Phys. Rev. **D51** (1995) 379.

[8] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Phys. Lett. **B245** (1990) 561, Nucl. Phys. **349** (1991) 727.

[9] N. Turok and J. Zadrozny, Phys. Rev. Lett. **65** (1990) 2331.

[10] M. Dine *et al.*, Phys. Lett. **B256** (1991) 86.

[11] L. McLerran *et al.* Phys. Lett. **B256** (1991) 451.

[12] L.B. Okun, *Leptons and Quarks* (North Holland Publishing Co., Amsterdam 1982)

[13] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Phys. Lett. **B263** (1991) 86.

[14] J. Ellis and M. Gaillard, Nucl. Phys. **B150** (1979) 141.

[15] I.B. Khriplovich and A.I. Vainshtein, Nucl. Phys. **B414** (1994) 27.

[16] E.P. Shabalin, Yad.Fiz. **28** (1978) 151 (Sov. J. Nucl. Phys. **28** (1978) 75).

[17] I.B. Khriplovich and M.E. Pospelov, Yad.Fiz. **53** (1991) 1030 (Sov.J.Nucl.Phys. **53** (1991) 638)

[18] C. Hamzaoui, A.I. Sanda, A. Soni, Phys. Rev. Lett. **63** (1989) 128.

[19] M. Peskin and T. Takeuchi, Phys. Rev. Lett. **65** (1990) 965, Phys. Rev. **D46** (1992) 381; G. Altarelli and R. Barbieri, Phys. Lett. **B253** (1991) 161; S. Bertolini and A. Sirlin, Phys. Lett. **B257** (1991) 179.

[20] S. Dawson and G. Valencia, Nucl. Phys. **348** (1991) 23; T.N. Truong, Phys. Rev. Lett. **70** (1993) 888.

[21] C. Hamzaoui and M.E. Pospelov, Phys. Lett. **B357** (1995) 616; Preprints BINP 95-59.

[22] A.D. Dolgov, Phys. Rep. **222** (1992) 309.

[23] M. Dine, Nucl. Phys. **B** (Proc. Suppl.) 37A (1994) 127.

[24] A.D. Sakharov, JETP Lett. 5 (1967) 24.

[25] P. Arnold and L. McLerran, Phys. Rev. **D36** (1987) 581.

[26] G. Anderson and L. Hall, Phys. Rev. **D45** (1992) 2685.

[27] I.B. Khriplovich and M.E. Pospelov, Nucl. Phys. **B420** (1994) 505.